Causal Inference for Beginners

02 Causal Graph

Representation of Causality

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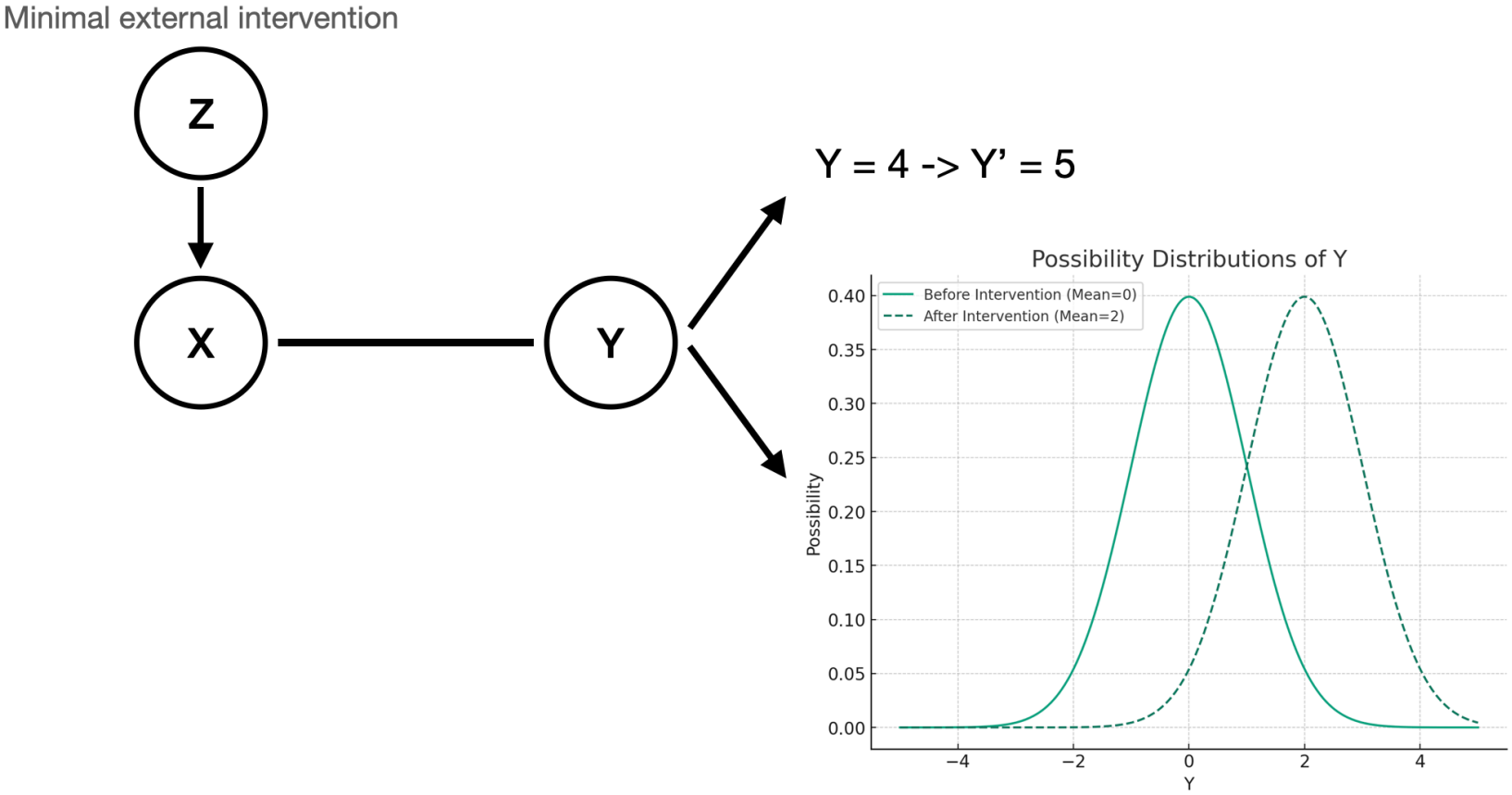
Outline

Review

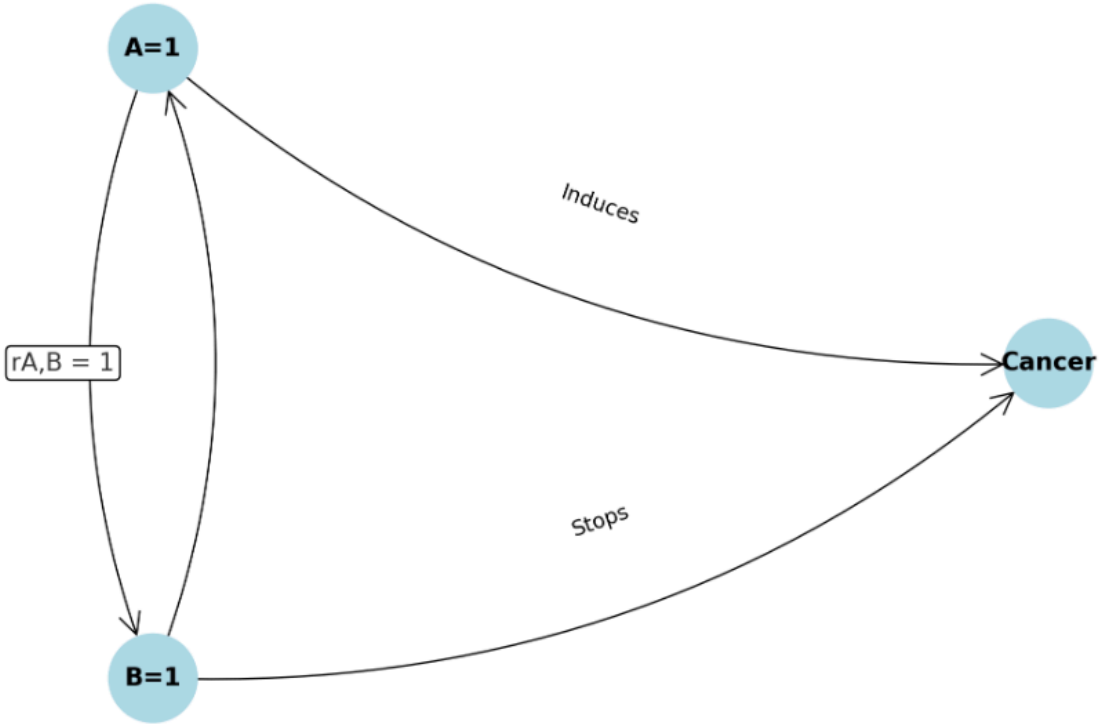
- Correlation is not causality
- Causality is about intervention



Causality

Review

- Does causality always mean correlation?
- EXP. Two genes (A and B) simultaneously influence cancer
  - A $= \{0, 1\}$ induces cancer
  - B $= \{0, 1\}$ stops cancer
  - either A and B are expressed at the same time or not ($r_{A,B} = 1$)



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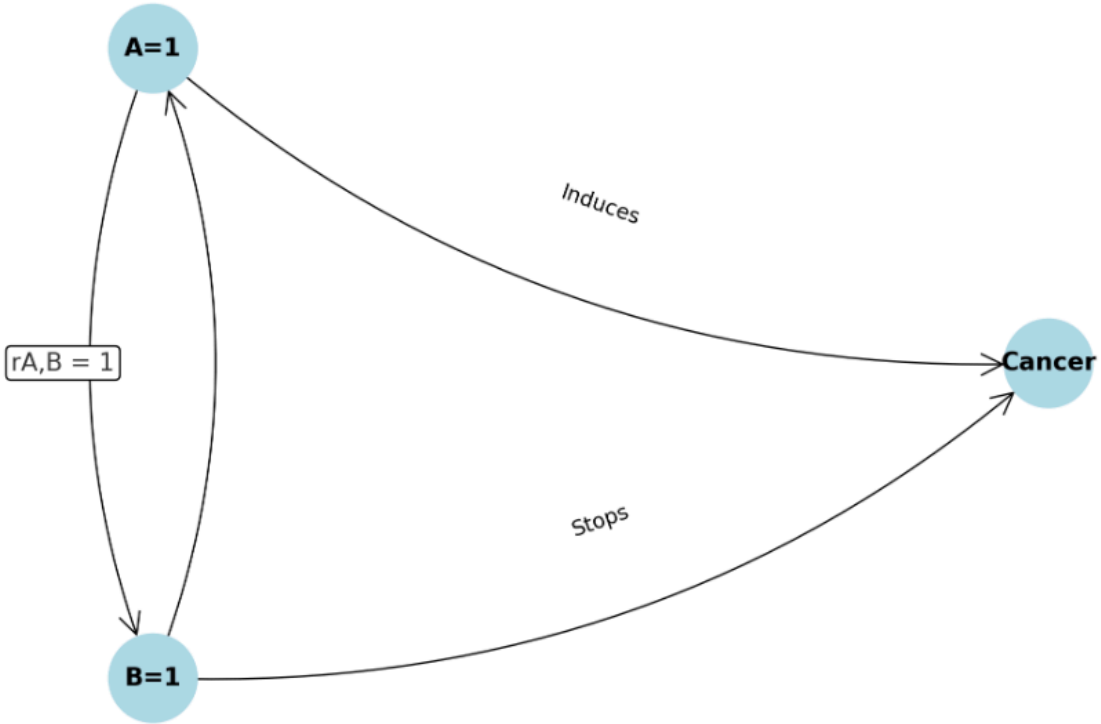
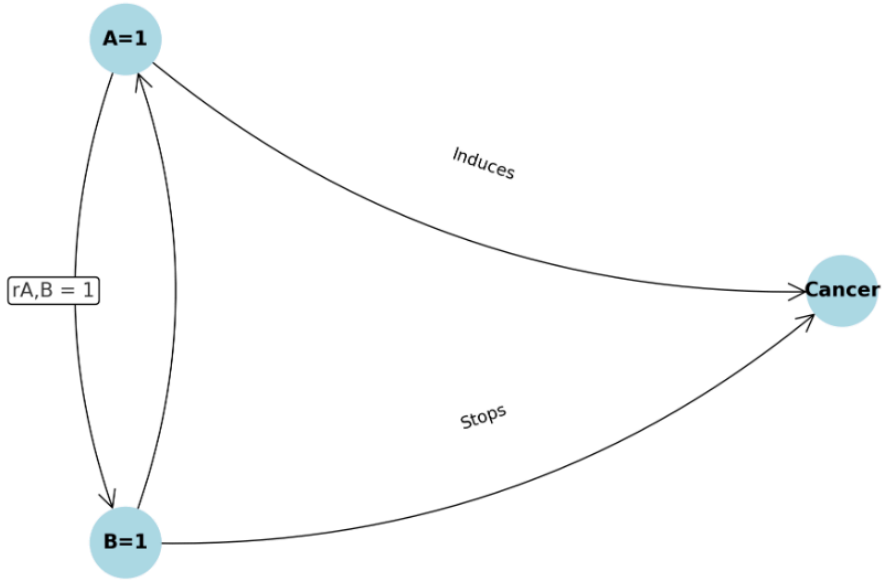


- Even though A is the cause of cancer, there is no correlation $(r_{A,Cancer} = 0)$

Introduction of Graph

- Causal graph model
  - using graph model to represent causal relationships
- Graph $G = (V, E)$
  - vertex (节点)
  - edge (边)
- Adjacent (邻接)
  - two nodes are connected by an edge
- Path (路径) between node X and Y
  - a sequence of nodes beginning with X and ending with Y
- Directed and undirected graph



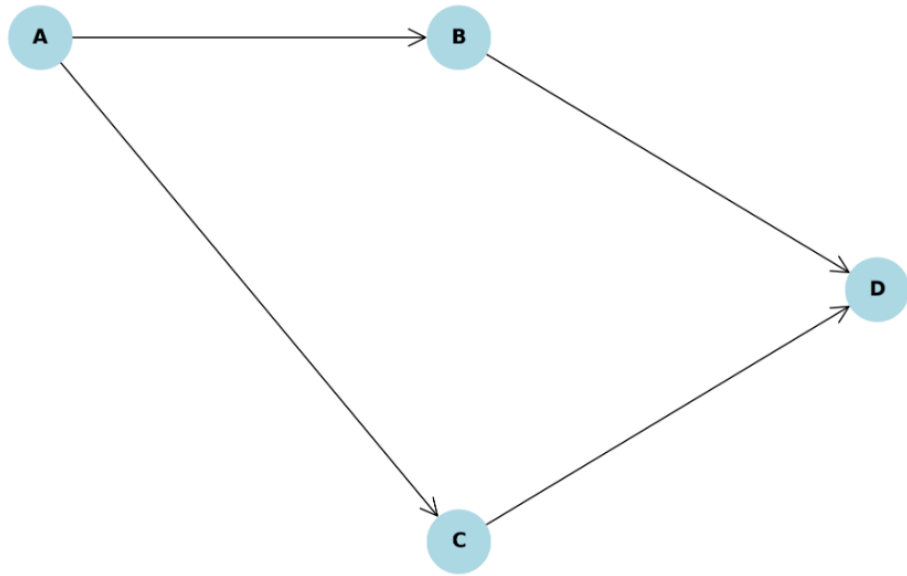
Example of graph

Directed Graph Undirected Graph

Directed and undirected graph

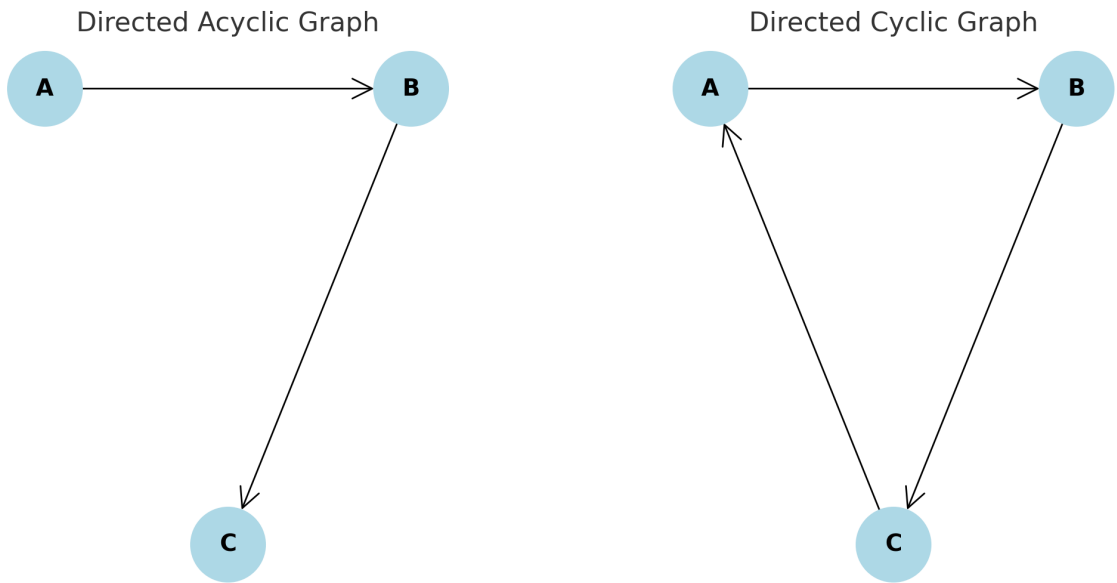
Introduction of Graph

- Parent (父节点) and ancestor (祖先)
  - parent: the beginning node of a directed edge ($parent(D) = \{B, C\}$)
  - ancestor: the beginning node of a directed path ($ancestor(D) = \{A, B, C\}$)
- Child (子节点) and descendent (后代)
  - child: the ending node of a directed edge ($child(A) = \{B, C\}$)
  - descendent: the ending node of a directed path ($descendent(D) = \{B, C, D\}$)



Introduction of Graph

- Cycle (环)
  - a directed path beginning and ending with the same node
- **Directed acyclic graph** (DAG, 有向无环图)
  - we focus on this type of graph
  - causal inference with directed cyclic graph (DCG) is still developing



Structural Causal Model (SCM)

- Structural Causal model (SCM)
  - describe causal relationships in a mathematical way
- Type of causal relation ($:=$)
  - direct cause:

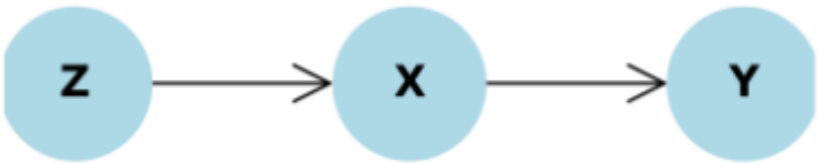
$$Y := f(X)$$

  - indirect cause:

$$Y := f(X)$$

$$X := g(Z)$$

$$Y := f(g(Z))$$



SCM

Structural Causal Model (SCM)

- Components of SCM
  - endogenous variables (内生变量, V): variables that we focus on
  - exogenous variables (外生变量, U): variables we don't know or do not matter (approximately random)
  - functions (F): mathematical functions describing causal relations
- EXP. Education and income
  - education (Edu) is determined by intelligence (Int, random variable) and other random factors (U_{Edu})

$$Edu = f_1(Int) + U_{Edu},$$

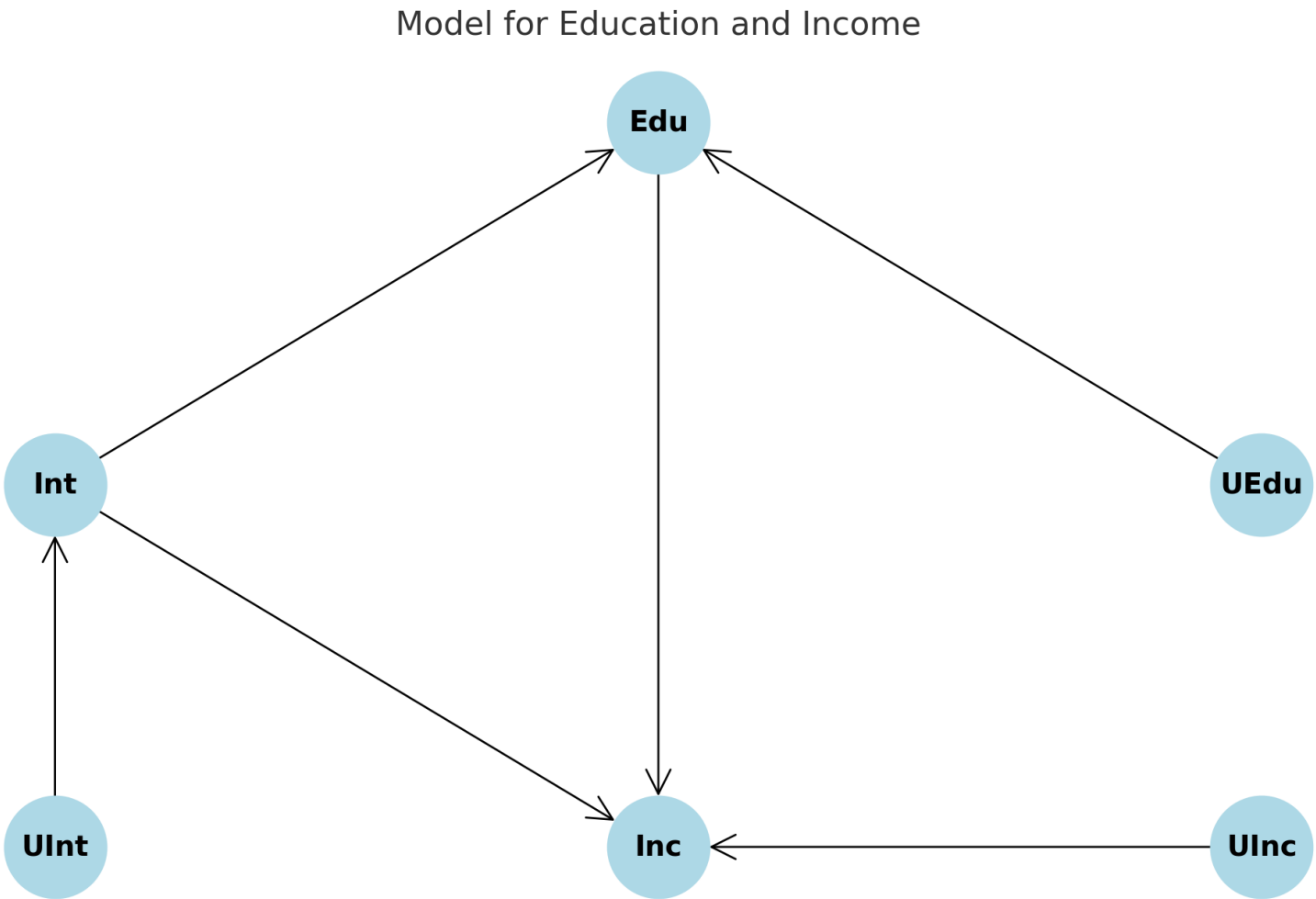
$$Int = U_{Int}$$

  - income (Inc) is determined by education, intelligence, and other random factors (U_{Inc})

$$Inc = f_2(Edu, Int) + U_{Inc}$$

Structural Causal Model (SCM)

- EXP. Education and income
  - in a graph way



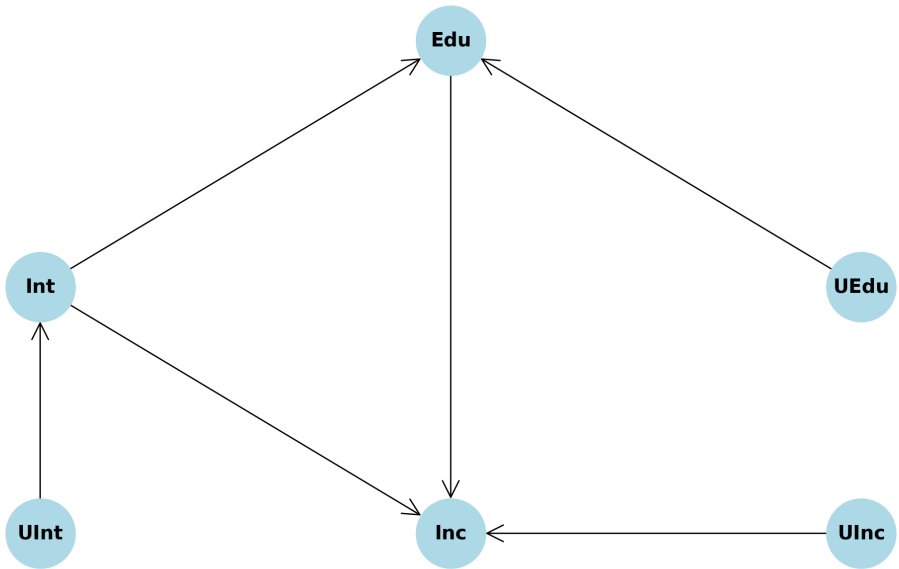
Structural Causal Model (SCM)

- The relation between SCM and Causal Graph
  - each SCM represents a causal graph, and a causal graph may represent various SCMs

The relation between SCM and Causal Graph

Graph	**SCM**
Root Nodes	Exogenous Variables U
Descendants of Root Nodes	Endogenous Variables V
Edges	Functions F

Model for Education and Income



Markov property of causal graph

- Anti-factual thinking (反事实, another definition of causality)
  - how does education influence people's income?
  - two parallel worlds (A and B) and two Jack living in these worlds ($Jack_A$ and $Jack_B$)
    - ★ $Jack_A$ finishes high school and goes to work
    - ★ $Jack_B$ finishes high school and continues to go to university
  - the difference between incomes of $Jack_A$ and $Jack_B$ in their 30^{th} birthday should be a clear effect of education

Markov property of causal graph

- Markov property of causal graph
  - Markov property (memory-less): the value of a variable in time $t+1$ is only determined by its value in time t (e.g., Brownian motion)
  - given $parent(Y)$, Y is independent of other non-descendants (or random)
  - in Jack's example, whether Jack goes to university or not is random
    - ★ $Jack_A$ and $Jack_B$ are totally the same when finishing high school
    - ★ in other words, Jack are given factors like family and intelligence
    - ★ $Jack_A$ flips a coin to go to work, $Jack_B$ flips a coin to go to university
- Rule of product decomposition (乘积分解法则)

$$P(X_1, X_2, ..., X_n) = \prod P(X_i \mid parent(X_i))$$

Conclusion

- some basic concepts of graph
  - vertex and edge
  - adjacent
  - path
  - directed graph
  - parent and ancestor, child and descendent
  - cycle
  - DAG
- SCM and its relation with graph
- Markov property of graph and how it makes things simple

Thx for listening!

Q & A